MINISTRY OF EDUCATION AND TRAINING

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DEVELOPMENT OF THE THIN-WALLED COMPOSITE BEAM MODEL UNDER MECHANICAL AND THERMAL LOADS

PH.D THESIS SUMMARY

MAJOR: ENGINEERING MECHANICS

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Declaration

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. I acknowledge the support I have received for my research through the guidance of Prof. Dr. Trung-Kien Nguyen and Dr. Do Tien Tho.

Xuan-Bach Bui

List of publications

ISI papers with peer-reviews:

1. **Bui, X.-B.**, T.-K. Nguyen, N.-D. Nguyen, and T.P. Vo, *A general higher-order shear deformation theory for buckling and free vibration analysis of laminated thin-walled composite I-beams.* Composite Structures, 2022. 295: p. 115775. https://doi.org/10.1016/j.compstruct.2022.115775
2. **Bui, X.-B.**, T.-K. Nguyen, and P.T.T. Nguyen, *Stochastic vibration and buckling analysis of functionally graded sandwich thin-walled beams.* Mechanics Based Design of Structures and Machines, 2023: p. 1-23. https://doi.org/10.1080/15397734.2023.2165101
3. **Bui, X.-B.**, T.-K. Nguyen, A. Karamanli, and T.P. Vo, *Size-dependent behaviours of functionally graded sandwich thin-walled beams based on the modified couple stress theory.* Aerospace Science and Technology, 2023. 142: p. 108664. https://doi.org/10.1016/j.ast.2023.108664
4. **Bui, X.-B**. and T.-K. Nguyen, *Deterministic and stochastic flexural behaviors of laminated composite thin-walled I-beams using a sinusoidal higher-order shear deformation theory.* Mechanics Based Design of Structures and Machines, 2023: p. 1-30. https://doi.org/10.1080/15397734.2023.2297840
5. **Bui, X.-B.**, P.T.T. Nguyen, and T.-K. Nguyen, *Spectral projection and linear regression approaches for stochastic flexural and vibration*

analysis of laminated composite beams. Archive of Applied Mechanics, 2024 .https://doi.org/10.1007/s00419-024-02565-x

Domestic papers with peer reviews and conference papers:

6. **Bui, X.B**., T.K. Nguyen, Q.C. Le, and T.T.P. Nguyen. *A novel two-variable model for bending analysis of laminated composite beams*. in *2020 5th International Conference on Green Technology and Sustainable Development (GTSD)*. 2020.
7. **Bui, X.-B.,** A.-C. Nguyen, N.-D. Nguyen, T.-T. Do, and T.-K. Nguyen, *Buckling analysis of laminated composite thin-walled I-beam under mechanical and thermal loads.* Vietnam Journal of Mechanics, 2023. 45(1): p. 75-90. https://doi.org/10.15625/0866-7136/17956
8. **Bui, X.-B**., T.-K. Nguyen, T.T.-P. Nguyen, and V.-T. Nguyen. *Stochastic Vibration Responses of Laminated Composite Beams Based on a Quasi-3D Theory*. in *ICSCEA 2021*. 2023. Singapore: Springer Nature Singapore.

Nomenclature

b, h, L: Width, height, length of the rectangular solid beam

u, w: axial and transverse displacements at any point on the rectangular solid beam

$\bar{u}, \bar{w}$: axial and transverse displacements at mid-plane of rectangular solid beams

$\mathbf{K}$: stiffness matrix

$\mathbf{M}$: mass matrix

$\mathbf{f}$: external force vector

b_1, b_2, b_3: the widths of the upper flange, web, and lower flange respectively of the I- or channel thin-walled beams

h_1, h_2, h_3: the thicknesses of the upper flange, web, and lower flange respectively of the I- or channel thin-walled beams

ϕ : rotational angle about the pole axis

E_1, E_2: Young's moduli

G_{12}, G_{23}, G_{13}: shear moduli

$\nu, \nu_{12}, \nu_{13}, \nu_{23}$: Poisson's ratios

ρ : mass density

$F_{\omega}(s)$: the warping function of thin-walled beams

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Abstract

Thin-walled beams are widely used in engineering fields like civil, aerospace, and automotive for their load capacity and lightness. This thesis investigates their structural responses, focusing on cross-section shapes, static analysis (deflection, buckling stability under thermal and mechanical loads), and vibration analysis (fundamental frequencies and mode shapes, particularly torsional modes for open-sections). It aims to enhance design, optimization, and safety in using advanced composite materials by predicting beam responses to various loads, material uncertainties, shear strain, and size effects. Previous models like Vlasov’s and first-order shear deformable beam theories are extended by proposing a high-order theory for composite beams. This model supports stochastic analysis (considering material property variations) and size-dependent effects analysis (using modified couple stress theory for microbeams). Techniques include a new beam solver, polynomial chaos expansion, and artificial neural networks for efficient and accurate response evaluation. Sensitivity analysis evaluates material property uncertainties' impact. The findings offer benchmarks for future research. Validation precedes these analyses, and MATLAB is used for all computations, prioritizing accuracy and efficiency.

1. Scope

1.1 Composite material

Composite materials have emerged as a core element in modern engineering and materials science, revolutionizing the way we design and manufacture a diverse range of structures and products. Unlike homogeneous materials, composites are fabricated by combining two or more distinct materials, each contributing its unique properties to create a synergistic material with enhanced characteristics. This blending of materials enables the development of materials that surpass the limitations of individual constituents, offering a remarkable balance of strength, stiffness, and versatility.

The state-of-the-art manufacturing techniques enable engineers to fabricate many kinds of composites. In the later sections, functionally graded composite (FGC), laminated composite (LC), and the porous metal foam are deeply analysed and discussed. These composites find applications in numerous fields, including aerospace, automotive, sport equipments, and structural engineering. They are particularly beneficial in components exposed to extreme conditions or varying loads, where a uniform material may not provide optimal performance.

1.2 Thin-walled beams

Thin-walled beams are structural elements characterized by having a relatively small ratio of wall thickness to their other dimensions, such as length and width, distinguising them with solid or thick-walled counterparts. The use and design of thin-walled beams is always driven by the need for structural efficiency, as the minimal use of material helps reduce weight while maintaining adequate strength and stiffness. For centuries, steel thin-walled beams have been used for building and bridges structures. Their behaviours and design are very well-studied as steel buildings constantly reach new heights and bridges keep increasing their span length. Nonetheless, when the newly introduced composite material are applied into thin-walled structures and the demand for structural efficiency grows, the research for composite thin-walled structures have a lot more gaps to fill. This study aims to analyse these composite thin-walled beam sections under mechanical and thermal loads.

1.3 Uncertainty quantification

In real-world scenarios, fluctuations in component materials due to production processes or unforeseen elements necessitate accounting for uncertainty to enhance beam response prediction reliability. Uncertainty quantification (UQ) addresses variability and imprecision in engineering models. Three approaches to UQ are utilized: Monte Carlo Simulation (MCS), Polynomial Chaos Expansion (PCE), and Artificial Neural Network (ANN).

MCS involves running numerous simulations with randomly generated input parameters, providing a distribution of possible beam response outcomes. While accurate, MCS can be computationally intensive. PCE and ANN offer more efficient alternatives, requiring fewer simulations to capture uncertainties and provide accurate predictions.

In addition to uncertainty quantification, sensitivity analysis examines the impact of each input parameter and their interactions on beam responses. Comparisons between MCS, PCE, and ANN are made based on the Sobol indices of beam simulations. Further details on these comparisons are discussed in subsequent chapters.

2. Theory overview

2.1 Solid beam theory

Composite solid beams have been applied in various engineering fields due to their advantages in versatility, strength and stiffness. Its properties can be engineerd to adapt to various requirements for the structure. Many beam models have been developed to accurately

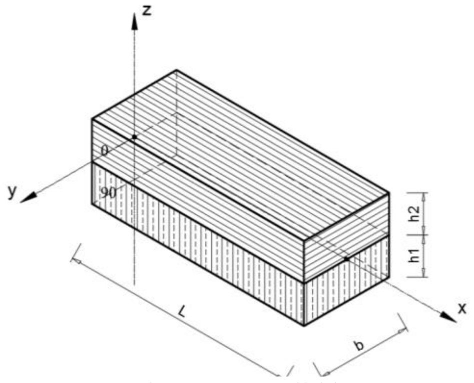

Figure: LC solid beam

predict the behavior of composite beams, which can be distinguised

between the following theoretical frameworks: Classical Beam Theory (CBT), First-Order Shear Deformation Theory (FOBT), Higher-Order Shear Deformation Theory with high-order variation of axial displacement (HOBT), and high-order theory approaching three dimensions with high-order changes of both axial and transverse displacements (quasi-3D).

-The Euler-Bernoulli beam theory: also known as the classical beam theory, assumes that the cross-section of the beam remains straight and perpendicular to the neutral axis before and after deformation. Based on this assumption, the displacement field is expressed as follows:

$$u(\mathrm{x},\mathrm{z},\mathrm{t}) = u_0(x,t) + z\theta_0(x,t) \quad \text{(1-1a)}$$

$$\mathrm{w}(\mathrm{x},\mathrm{z},\mathrm{t}) = \mathrm{w}_0(x,t) \quad \text{(1-1b)}$$

where $\mathrm{u}_0, \mathrm{w}_0$ are the axial displacement and transverse displacement at the beam’s neutral axis. This theory overestimates the stiffness of the beam and its applicability is restricted to slender beams with large length-to-depth ratio.

-The Timoshenko beam theory: addresses some of the limitations inherent in the Euler-Bernoulli beam theory. Timoshenko beam theory takes into account the effects of shear deformation and rotational inertia. This makes it more accurate for a wider range of beams, especially those that are short, thick, or subjected to high-frequency loading. The displacement field is given as:

$$u(\mathrm{x,z,t}) = u_0(x,t) + z\theta_0(x,t) \quad \text{(1-2a)}$$

$$\mathrm{w}(\mathrm{x,z,t}) = \mathrm{w}_0(x,t) \quad \text{(1-2b)}$$

where θ_0 is the rotation angle of the cross-section relative to the perpendicular axis. The Timoshenko beam has three variables and accounts for the effect of transverse shear deformation, thus providing a more appropriate prediction of beam behavior compared to the Euler-Bernoulli beam. However, because the transverse shear deformation is constant along the length of the beam, this leads to an unrealistic distribution of shear stress. Therefore, a shear correction factor is added to adjust the calculation of the shear force, and a factor of 5/6 is commonly used. In practice, this beam theory has been applied in the majority of commercial software.

-The Higher-Order shear deformation theory: includes higher-order terms in the displacement field equations, allowing for a more accurate representation of the shear deformation throughout the depth of the beam. This is crucial for accurately predicting the behavior of thick beams, composite beams, and beams made of materials with a low modulus of elasticity. The displacement field is as follows:

$$u(\mathrm{x},\mathrm{z},\mathrm{t}) = u_0(x,t) - \mathrm{z}\,\mathrm{w}_{0.x} + f(z)\theta_0(x,t) \qquad (1\text{-}3a)$$

$$\mathrm{w}(\mathrm{x},\mathrm{z},\mathrm{t}) = \mathrm{w}_0(x,t) \qquad (1\text{-}3b)$$

where $f(z)$ is the high-order shear function. This function has been proposed by many past authors based on the following conditions: $f^{'}(z = \pm h/2) = 0$ and $f(z)$ must be continuous in the z domain.

-The Quasi-3D beam theory: bridge the gap between two-dimensional beam theories and fully three-dimensional elasticity solutions. Unlike the aforementioned beam theories, which simplify the stress and strain within the beam to a one- or two-dimensional problem, quasi-3D beam theory incorporates aspects of three-dimensional stress and strain. This approach allows for a more accurate representation of the physical behavior of beams, including the effects of lateral strains and out-of-plane deformations. The displacement field contains four variables $u_0, \mathrm{w}_0, \theta_0, \mathrm{w}_{z0}$ to be solved:

$$u(\mathrm{x},\mathrm{z},\mathrm{t}) = u_0(x,t) - \mathrm{z}\,\mathrm{w}_{0.x} + f(z)\theta_0(x,t) \qquad (1\text{-}4a)$$

$$\mathrm{w}(\mathrm{x},\mathrm{z},\mathrm{t}) = \mathrm{w}_0(x,t) + \mathrm{g}(\mathrm{z})\mathrm{w}_{z0}(x,t) \qquad (1\text{-}4b)$$

It is worth noting that the more complex the theory is, the more versatile and accurate the beam analysis becomes. The better accuracy comes with the increased cost of computational resource.

2.2 Thin-walled beam theory

In the 2000s-2010s, the researches delved into material optimization and advanced manufacturing techniques for thin-walled composite beams. Thostenson et al. [10] gave a review on advances in the science and technology of carbon nanotubes and their composites. Gay and Suong [11] focused on optimizing the design and manufacturing of thin-walled composite beams to achieve better performance and efficiency. Librescu and Song [12] contributed greatly to the theory and applications of thin-walled composite beams. Vo et al. [13] developed the finite element model for vibration and buckling of functionally graded sandwich beams based on a refined shear deformation theory. Nguyen et al. [14] proposed a new trigonometric-series solution for analysis of laminated composite beams. Lee et al. [15-18] contributed many analyses for thin-walled composite beams This period also saw advancements in material science, leading to the development of new composite materials with enhanced properties.

In recent years (2020s), research has been exploring more sophisticated areas such as the use of nano-materials in composites, the stochastic behaviours of composites, and smart composite materials that can adapt to changing conditions. For nano- and micro-structures, Ghane et al. [19] studied the vibration of fluid-conveying nanotubes subjected to magnetic field based on the thin-walled Timoshenko beam theory. Xie et al. [20] exprimented and modelled the vibration of multi-scale sandwich micro-beams. ND Nguyen et al. [21] investigated the LC micro-beam based on the modified couple stress theory using a Ritz type solution with exponential trial functions. These current topics in the researches of composite thin-walled structures are the pillars of this thesis.

Based on the definition of Vlasov [23], thin-walled beams are beams with $\frac{h}{l} \le 0.1$ and $\frac{l}{L} \le 0.1$, where h is the wall thickness, l is any characteristic dimension of the cross-section, and L is the beam length. The wall thickness can only vary along the beam's cross section contour, but remains constant along the beam span. A same set of coordinates for the analysis of thin-walled beams is used throughout this thesis. Cartesian coordinate system (x, y, z), local plate coordinate system (n, s, z) and contour coordinate s along the profile of the section are considered. It is assumed that θ is an angle

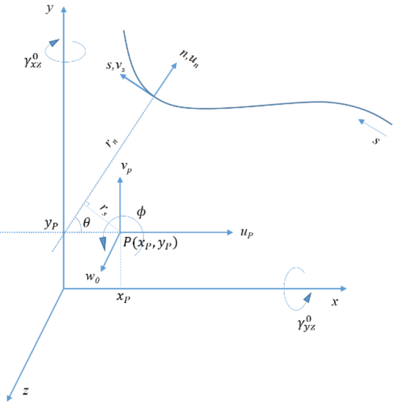

Figure: Thin-walled coordinate systems

of orientation between (n, s, z) and (x, y, z) coordinate systems, the pole P with coordinates (x_P, y_P) is the shear center of the section.

2.3 Composite materials' constitutive relations

There are three main types of composote materials used in this thesis: laminated composite material, functionally graded material, and porous metal foam material. The effects of anisotropy in these composite materials allow designer to efficiently aligning the material's structure with the load paths, therefore, reducing structures' weight without compromising strength. These effects are described through the constitutive relation equations shown below.

Based on Hooke’s law, the stress-strain relations for anisotropic materials can be expressed in matrix form as follows:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix} \tag{1-8}$$

where σ_{ij} are the stress components, ε_{ij} are the axial strain components, and γ_{ij} are the shear strain components.

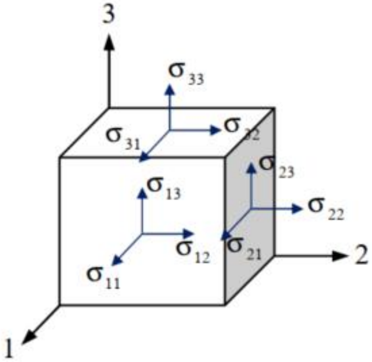

Figure: Stress and strain component in the coordinate system

2.4 Stochastic analysis

Stochastic analysis is a branch of computing science and mathematics that study the randomness in a system or process. It is widely used in many fields such as finance, physics, biology and engineering. The theory foundation of this study branch trace back to Norbert Wiener's work in the 1940s, famously called the Wiener process, that investigate the one-dimensional Brownian motion. This theory introduced the idea that a stochastic process could be decomposed into a series of orthogonal polynomial functions of random variables, and has been deeply discussed by Szabados [24]. With the advances of the computing systems in the 1990s, Ghanem and Spanos presented the use of polynomial chaos for arbitrary distributions. Andrew and Askey [25] contributed to the polynomial chaos theory by proposing the Askey scheme which organises the orthogonal polynomials and expands the understanding of the stochastic processes. From the 2000s to present, the increasing computational resources and state-of-the-art numerical algorithms have been propelling the use of the polynomial chaos expansion (PCE) across various fields. [26-28]

In this thesis, the stochastic analysis is applied for thin-walled composite beams to study how the random variables of the beam's parameters affect the beam's mechanical responses. These three techniques are depicted briefly below and more detailed in the chapter Four: Monte Carlo Simulation (MCS) , Polynomial Chaos Expansion (PCE), and Artificial Neural Network (ANN).

-Monte Carlo Simulation (MCS): the MCS is simply a technique that generate random input samples, and observe how the randomness impacts the outputs. These inputs and outputs can be probabilistic, and are subjected to the physical nature of the variables. Among the 3 techniques, MCS is the simplest in term of mathematical formulation, and can give the most accurate representation of the beams' outcomes. However, the accuracy can come with an expensive computing cost, especially when the number of random input variables and samples increases. The PCE and ANN are modern techniques that help overcomes the MCS's problem while achieving the same level of accuracy.

-Polynomial Chaos Expansion (PCE): the PCE represents the uncertain parameters in a system as a series of polynomial of random variables multiplied by respective coefficients. These polynomials are chosen to be orthogonal with respect to the probability distribution of the inputs.

$$\hat{u} \simeq \hat{u}_{PCE}\left(\mathbf{x}\right) = \sum_{i=0}^{P-1} c_i He_i\left(\mathbf{q}\right) + \varepsilon \qquad (1\text{-}21)$$

where responses $\hat{u}$ of FG sandwich thin-walled beams $\hat{u}_{PCE}$ are the responses of interest obtained from PCE; $\mathbf{q}$ is a vector of independent random variables in the PCE space mapped to physical random parameters $\mathbf{x}$; He_i are multivariate orthogonal basis functions, which depends on the probability distribution of the random inputs; c_i are coefficients to be determined so that the residual ε is minimized

In this thesis, the residual ε is minimized by the means of least-square regression and Gaussian quadrature method.

-Artificial Neural Network (ANN): ANNs are computational models inspired by the human brain, used extensively in machine learning. As shown in the Fig. below, there are three main types of layers in ANN: Input layer, Hidden layer, and Output layer. The hidden layers are where most of the

Even though the ANN has been recently applied for the behaviour prediction of thin-walled beams [29-31], there is no research that conduct a thorough comparison between MCS, PCE, and ANN for the stochastic analysis of thin-walled composite beams. This gap is considered in the latter chapter of this thesis.

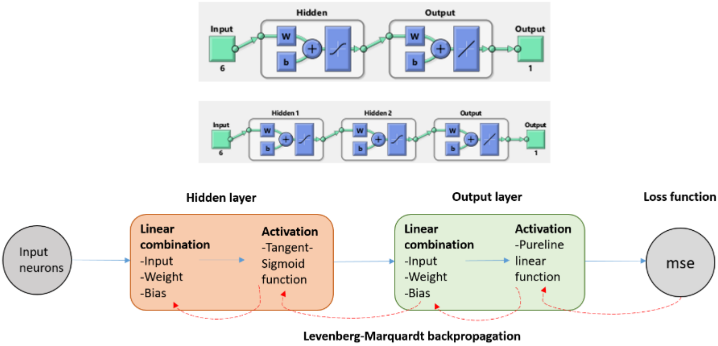

Figure: Artificial neural network workflow

3. Research objectives

The research objectives for this PhD thesis is the study of composite thin-walled beams using MATLAB and higher-order shear deformable theory, coupled with the uncertainty quantification, size-dependent effects on micro-beams, and thermal buckling phenomena, are outlined as follows:

- To develop and validate a computational framework: to implement higher-order shear deformable theory for the analysis of composite thin-walled beams, including laminated, functionally graded, and porous metal foam beams. This framework aims to accurately predict their mechanical behavior under various loading conditions.
- To investigate the impact of material heterogeneity: Study how variations in material composition and distribution within functionally graded, laminated composite, and porous metal foam beams influence their mechanical properties and overall structural performance, using the developed computational model.
- To quantify uncertainties: Employ probabilistic methods to quantify uncertainties related to material properties, geometric imperfections, and loading conditions, and assess their impact on the behavior of composite thin-walled beams.
- To examine the size-dependent effects on micro-beams: Explore the size-dependent behavior of micro-beams made from composite materials, thereby enhancing the understanding of scale effects in micro-scale structures.

- To analyze thermal buckling behavior: Investigate the thermal buckling characteristics of composite thin-walled beams under various thermal loading scenarios. This includes understanding the role of temperature gradients and thermal loads on the stability of these thin-walled beams.
- To validate models through comparative analysis: Compare the predictions of the developed models with existing analytical, numerical, and experimental results from the literature, to validate the accuracy and reliability of the proposed computational framework.

By achieving these objectives, this thesis aims to advance the state of knowledge in the field of composite thin-walled beams, providing a robust computational tool for other researchers, and proposing the optimization approaches for these critical structural elements.

4. Research method

In each of the following chapters, the data are presented in the order based on the research method. The thin-walled beam models and simulations are verified with multiple notable articles and experimental results. Subsequently, the parametric studies are conducted with a range of input parameters fed into this thesis’s simulation models. This sequence of research method ensures the accuracy and validity of the new contributions.

5. Conclusion

In conclusion, this thesis has delved into the comprehensive study of thin-walled beams, emphasizing their widespread applications in diverse engineering domains such as civil, aerospace, and automobile engineering due to their notable advantages in load-carrying capacity and lightweight properties. The investigation primarily focused on understanding the structural responses of thin-walled beams with different cross-section shapes, addressing the static, buckling and dynamic responses to mechanical and thermal loads.

For static analysis, the accurate prediction of beam deflection and buckling stability under varied thermal and mechanical loading conditions is crucial in the design process. Additionally, the thesis explored vibration analysis, scrutinizing the free vibration fundamental frequencies and mode shapes, with a particular emphasis on torsional modes for open-section beams. Building upon established thin-walled beam models, such as Vlasov's model and first-order shear deformable thin-walled beam theory, this thesis proposed a general high-order shear deformable beam theory tailored for thin-walled composite beams. The general high-order shear deformable thin-walled beam theory can better predict the responses of thin-walled beams with low length-to-depth ratio.

The exploration extended to stochastic analysis, where the inherent randomness in constituent material properties was considered. This led to the development of a novel beam solver employing hybrid series-type approximation functions, coupled with the utilization of polynomial chaos expansion (PCE) and artificial neural network (ANN) techniques for efficient evaluations of stochastic responses. The findings, benchmarked against crude Monte Carlo simulation, provided valuable insights into the impact of material property uncertainties on stochastic responses, serving as potential benchmarks for the scientific and engineering community. The PCE and ANN employed for thin-walled composite beams in this thesis requires only 256 training samples and much less computational time compared to 10^6 samples of MCS benchmark while maintaining the predicted output accuracy.

Furthermore, the thesis carried out the size-dependent effects analysis, replacing classical continuum mechanics with the modified couple stress theory. Numerical results elucidated the influences of material distribution, span-to-height ratio, and material length scale parameters on the bending and vibration behaviors of microbeams under various boundary conditions. The micro-beam exhibits much stiffer behaviours compared to its macro- counterpart.

Prior to conducting these analyses, rigorous validation of the theoretical and numerical models was performed, ensuring the accuracy, efficiency, and computing time of the analysis software code. The culmination of these studies contributes not only to the theoretical advancements in thin-walled beam modeling but also establishes benchmark results for future scientific and engineering research in this domain. The consideration of efficiency and computing time underscores the practical significance of the findings by the PhD candidate for the broader engineering community.

6. Future directions

The phD candidate plans the following directions for the future researches:

- The thermal and hygro analysis of closed-section thin-walled beams with arbitary cross section shapes
- The incorporation of isogeometric method to replace the Ritz solution in this thesis
- The effects of geometrical nonlinearity on thin-walled beams
- The effects of other loading conditions (moving loads, impact loads) on thin-walled beams
- The Karhunen-Loeve expansion for the stochastic analysis of thin-walled beams
- Artificial neural network’s hyperparameter optimisation

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